

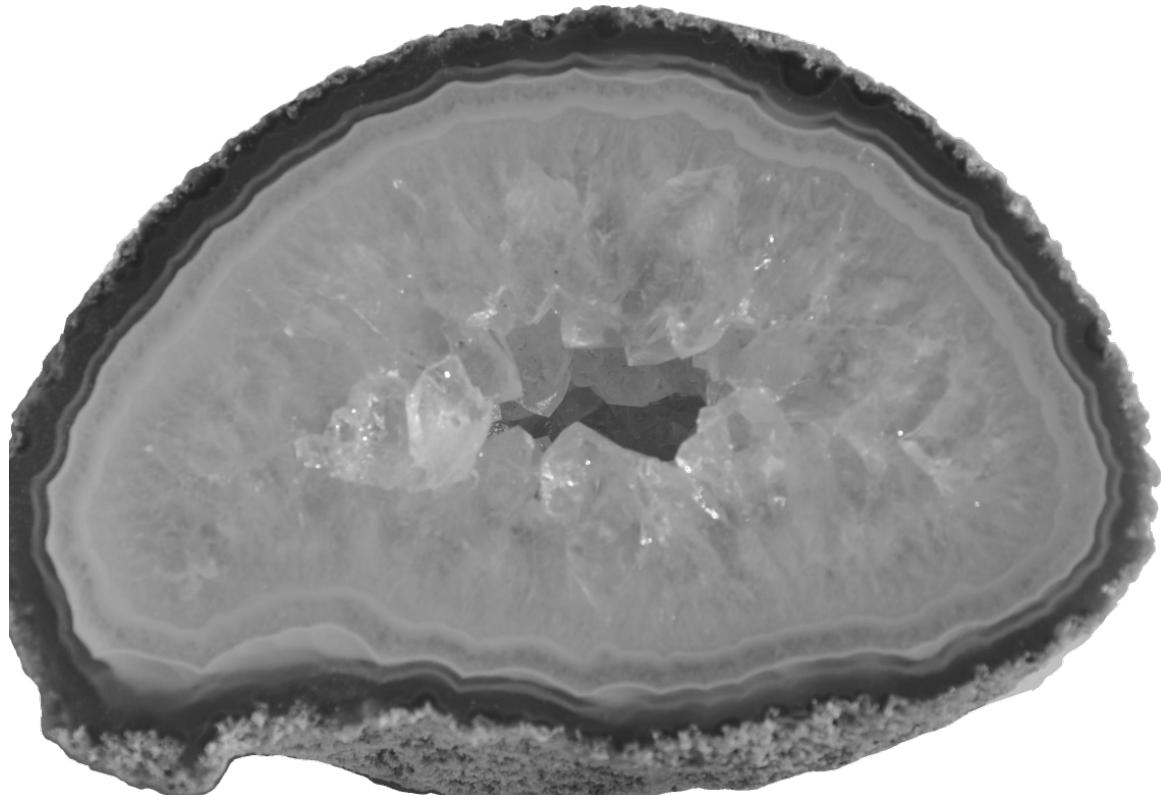
# AGATES Workshop

## Geometry of Secants

October 24<sup>th</sup>–28<sup>th</sup>, 2022

Book of Abstracts

<https://agates.mimuw.edu.pl/index.php/agates/geometry-of-secants>



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# Schedule

All lectures in room 321 on the third floor.

	Mon, Oct 24	Tue, Oct 25	Wed, Oct 26	Thu, Oct 27	Fri, Oct.28	
9:00–10:00	Kapustka	Han	Choe	Postinghel	Casarotti	
10:00–10:30	coffee break (room 409)					
10:30–11:30	Di Rocco	Ranestad	Ottaviani	Ciliberto	Abo	
11:40–12:40	Gimigliano	Laface	Torrance	Massarenti	Vannieuwenhoven	
12:40–14:30	lunch (not organised)					
14:30–15:30	working group	working group	free afternoon	working group	working group	
15:30–16:00	coffee break (409)			coffee break (409)		
16:00–17:00	working group	working group		working group		
18:00–…		dinner on the ground floor				

# Titles and Abstracts

In order of appearance in the Programme of the Workshop

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## Quaternary quartic forms and Gorenstein rings

Mon Oct 24  
9:00

Michał Kapustka

Institute of Mathematics of Polish Academy of Sciences, Kraków, Poland

A quartic form in four variables by apolarity gives rise to an Artinian Gorenstein ring of regularity and codimension equal to 4. We will investigate the relation between the rank and powersum decompositions of the quartic form and the betti table of the associated Artinian Gorenstein ring. We will then focus on describing the geometry of the corresponding varieties of sums of powers. If time permits, we will also provide some explicit constructions of the Artinian Gorenstein rings and discuss their liftings to higher dimensions.

This is joint work with G. Kapustka, K. Ranestad, H. Schenck, M. Stillman, B. Yuan.

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## Higher order bottlenecks

Mon Oct 24  
10:30

Sandra Di Rocco

KTH Royal Institute of Technology, Stockholm, Sweden

Constructing effective samples on an algebraic variety whose Vietoris Rips complex recovers the topology of the underline space is an important line of research in algebraic data analysis. I will introduce the concept of higher order bottlenecks which are invariants governing the density of such samples. The theory of secants plays an important role in proving that general complete intersections have finitely many bottlenecks of order less than the dimension of the ambient space.

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## Peculiar 0-dimensional schemes in the plane

Mon Oct 24  
11:40

Alessandro Gimigliano

University of Bologna, Italy

We will consider certain 0-dimensional schemes supported at a point in the plane which satisfy symmetry conditions. We will consider applications to Segre-Veronese surfaces.

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## Singular points from subsecant loci of $k$ -secants of Veronese varieties

Tue Oct 25  
9:00

Kangjin Han

Daegu-Gyeongbuk Institute of Science & Technology

For a projective variety  $X$  in  $\mathbb{P}^N$ , the  $k$ -secant variety  $\sigma_k(X)$  is defined to be the closure of the union of  $k$ -planes in  $\mathbb{P}^N$  spanned by  $k$ -points of  $X$ . In this talk, we consider the singular loci of higher secant varieties of the image of the  $d$ -uple Veronese embedding of projective  $n$ -space,  $\nu_d(\mathbb{P}^n)$ . A complete description of singular locus of  $k$ -secant of  $\nu_d(\mathbb{P}^n)$  has been known only for  $k \leq 3$ . First, we review some basic notions and some related results. I explain our new technique which investigates geometry of moving tangents along subvarieties in projective space. Using this, we determine some new singular points coming from 'subsecant loci' of  $k$ -secant of  $\nu_d(\mathbb{P}^n)$  for arbitrary  $k$ . We also report a result on the singular locus of 4th-secant of Veronese varieties. In the end, we discuss some generalizations and subsequent questions for further exploration.

This is a joint work with Katsuhisa Furukawa.

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## Bad limits on power sum varieties

Tue Oct 25  
10:30

Kristian Ranestad

University of Oslo, Norway

The variety of sums of powers,  $\text{VSP}(F, r)$  of a homogeneous form  $F$  of rank  $r$  is the closure in the Hilbert scheme of apolar schemes of length  $r$ . A bad limit is a scheme in the closure that is not apolar to  $F$ . I will discuss examples of bad limits, including examples for quadrics found by Joachim Jelisiejew that contradicts earlier results on polar simplicies.

This is report on work in progress with Jelisiejew and Schreyer and with Grzegorz and Michał Kapustka.

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## On secant defectiveness and identifiability of Segre-Veronese varieties

Tue Oct 25  
11:40

Antonio Laface

University of Concepción, Chile

For Segre-Veronese varieties the problem of secant  $h$ -defectiveness has been solved in some very special cases, mostly for products of few factors, for small values of  $h$  and it is completely solved for Segre-Veronese products of projective lines. Our main result provides an, almost asymptotically sharp, upper bound on  $h$  for non  $h$ -defectiveness of Segre-Veronese varieties. To prove it we developed an algorithm, based on a convex geometry translation of Terracini's lemma, which provides a lower bound for the dimension of the  $h$ -secant variety of a toric variety.

This is joint work with Alex Massarenti and Rick Rischter.

## Parallel syzygies of higher secant varieties

Wed Oct 26  
9.00

Junho Choe

Korea Institute for Advanced Study (KIAS), South Korea

This talk will deal with parallel phenomena that equations, relations and higher syzygies of higher secant varieties show. We will look at three subjects closely related to each other. Some of contents are based on joint works with Prof. Sijong Kwak.

First, we will discuss syzygies of higher secant varieties of minimal degree and del Pezzo higher secant varieties. The corresponding classical fact is that a variety of minimal degree (resp. del Pezzo variety) has unique Betti table according to its codimension.

Second, we will mention Ein-Niu-Park's generalized  $(2g+1+p)$ -theorem for higher secant varieties to smooth curves. The original  $(2g+1+p)$ -theorem due to Green says that any completely embedded smooth curve of degree  $2g+1+p$  satisfies property  $N_p$ .

Last, ongoing research will be introduced, namely a generalized gonality conjecture for higher secant varieties to smooth curves. Introduced by Green and Lazarsfeld, and proved by Ein and Lazarsfeld, the gonality conjecture reads off the gonality of a smooth curve from syzygies of the curve completely embedded with high degree.

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## Sums of k-th powers of homogeneous polynomials

Wed Oct 26  
10.30

Giorgio Ottaviani

University of Florence, Italy

When  $k$  is a divisor of  $d$ , expressing a complex homogeneous polynomial of degree  $d$  as sum of  $k$ -th powers generalizes the classical Waring decomposition, which is the case  $k=d$ . We discuss the main features of this problem, regarding existence and uniqueness of the decomposition, with special emphasis on the case  $k=2$  (sum of squares).

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## Induction and secant varieties to Chow varieties

Wed Oct 26  
11:40

Douglas Torrance

Piedmont University, Demorest, GA, USA

With the exception of secant varieties to varieties of completely decomposable quadrics, it is conjectured that all secant varieties of Chow varieties are nondefective. We discuss the induction techniques that led to the proofs of several of the known cases.

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Thu Oct 27  
9:00

## Waring Identifiability for forms of degree $kd$

Elisa Postinghel  
University of Trento

An upper bound on the rank of the decompositions of general forms of degree  $kd$  as a sum of  $k$ th powers of degree  $d$  forms was proposed by Fröberg, Ottaviani and Shapiro. An approach to the identifiability problem for these decompositions will be discussed in this talk. This will employ methods from classical algebraic geometry (Terracini's style) together with certain (toric) degeneration arguments. This is joint work (in progress) with Alex Casarotti.

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Thu Oct 27  
10:30

## On defective 4-folds

Ciro Ciliberto  
University of Rome Tor Vergata, Italy

I will talk about the classification of defective 4-folds. This is joint work with L. Chiantini and F. Russo.

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Thu Oct 27  
11:40

## Bronowski's conjecture and the identifiability of projective varieties

Alex Massarenti  
University of Ferrara, Italy

Let  $X$  be an irreducible variety of dimension  $n$  embedded in a projective space of dimension  $hn + h - 1$ . Bronowski's conjecture predicts that  $X$  is  $h$  identifiable if and only if the general  $(h - 1)$ -tangential projection of  $X$  is birational. We will discuss several aspects of this conjecture and show how it can be used to relate the notion of defectiveness to that of identifiability of  $X$ .

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## **Identifiability for powers of quadratic polynomials**

Fri Oct 28  
9.00

Alex Casarotti

University of Trento, Italy

The generalization of classical polynomial identifiability to identifiability of forms of degree  $kd$  as sums of  $k$ -th powers of forms of degree  $d$  was investigated first by Fröberg, Ottaviani and Shapiro. Using classical computational arguments of the tangential contact locus we will improve their bounds for the case of powers of quadratic polynomials, i.e.  $d = 2$ . We will also show the complete identifiability behaviour for quadratic polynomials in  $n$  variables, where  $n < 16$ . Finally we will apply our results to the study of particular mixtures of centered Gaussians.

This is a joint work with Alexander Taveira Blomenhofer, Alessandro Oneto and Mateusz Michałek.

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## **Secant varieties of Segre-Veronese varieties**

Fri Oct 28  
10:30

Hirotachi Abo

University of Idaho, Moscow, ID, USA

The purpose of this talk is to discuss the current status of the classification problem for defective Segre-Veronese varieties.

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## **Algebraic compressed sensing (of secant varieties)**

Fri Oct 28  
11:40

Nick Vannieuwenhoven

KU Leuven, Belgium

In this talk, I introduce the broad class of algebraic compressed sensing problems. Herein, the goal is to recover a point on an algebraic variety from a small number of linear projections. This includes, among others, the low-rank matrix and tensor completion problems. We employ techniques from algebraic geometry to answer basic questions related to this problem, such as the existence, local recoverability, global uniqueness / identifiability, and local smoothness of solutions of generic compressed sensing problems.

I will also discuss partial results for the specific, non-generic compressed sensing problem of coordinate projections of low-rank matrices (i.e. points on secant varieties of two-factor Segre varieties). The talk will be concluded with interesting open problems about the identifiability of coordinate projections of secant varieties of arbitrary varieties.

This is joint work with Paul Breiding, Fulvio Gesmundo, and Mateusz Michałek.

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