AGATES KICKOFF WORKSHOP

Plenary Talks

Hirotachi Abo

Title. *Eigenconfigurations of tensors: recent developments*

Abstract. An eigenconfiguration of a tensor is the algebraic set obtained as the determinantal locus of a certain matrix associated with the tensor. Several different aspects of eigenconfigurations of tensors were explored in the paper entitled "Eigenconfigurations of tensors," where the authors suggested several different problems. The purpose of this talk is twofold: the first is to summarize recent progress towards these problems and the second is to discuss future challenges.

Alessandra Bernardi

Title. *On the regularity of certain 0-dimensional schemes*

Abstract. Given a homogeneous polynomial, there is a natural way of building apolar schemes after having fixed the supports. I will discuss the problem of studying any regularity of those schemes.

Maria Chiara Brambilla

Title. *On the dimension of linear systems with multiple base points*

Abstract. The study of linear systems of hypersurfaces in complex projective spaces with finitely many multiple general base points is a classical problem in Algebraic Geometry. It is related with polynomial interpolation, with the Waring problem, with the classification of defective higher secant varieties and with the rank of tensors.

Even if linear systems have been studied for more than a century, basic questions are still open in general. A linear system is called special if its dimension is not the expected one. Complete classification results are known only in few cases. The most important result in this field is the Alexander-Hirschowitz theorem which concerns double points. For any multiplicity the Segre-Harbourne-Gimigliano-Hirschowitz conjecture, in the case of the plane, and the Laface-Ugaglia conjecture, in the case of $\mathbb{P}^3$, are still open.

In my talk I will focus on two different directions of research: (1) linear systems with double points in projective varieties such as Segre and Segre-Veronese varieties; (2) linear systems with multiple points in projective spaces and a general study of the obstructions which produce speciality.

The first topic is strictly related to the rank of tensors and partially symmetric tensors, while the second one is connected with problems in Birational Geometry and Mori Dream Space theory.

Christian Ikenmeyer

Title. *De-bordering symmetric border rank*

Abstract. We discuss the gap between Waring rank and border Waring rank from an algebraic complexity theory viewpoint.

Khazhgali Kozhasov

Title. *Real aspects of the problem of rank-one approximation*

Abstract. A best rank-one approximation to a real tensor T is a rank-one tensor T_1 that minimizes (the Frobenius) norm $\|T - S_1\|$ among all rank-one tensors S_1 of a given format. T_1 is, in particular, a critical point of the square of the distance function $dist_T : S_1 -> \|T - S_1\|^2$ on the variety X of rank-one tensors. The largest possible number N of critical points of $dist_T$ among all generic T can be interpreted as a measure of complexity of the rank-one approximation problem. I will discuss a bound on N due to Friedland and Ottaviani and pose a question regarding optimality of this bound. I will explain a technique that has been used to solve the analogous problem for the case of symmetric tensors and symmetric rank-one approximations. Furthemore, I will mention a similar problem in algebraic statistics that concerns rank-one approximations of probability tensors and state a recent related conjecture by Boege, Petrović and Sturmfels.

Mario Kummer

Title. *Secant varieties of real curves*

Abstract. I will report from a joint work with Rainer Sinn on the real geometry of (higher) secants of real curves. I will explain applications to the theory of convex optimization and highlight connections to knot theory. Then I discuss possible generalizations to varieties of higher dimension and pose open problems.

Benjamin Lovitz

Title. *New techniques for bounding stabilizer rank*

Abstract. It is a major open problem in quantum information to determine which quantum computations can be efficiently simulated by classical computers. In a similar spirit to how the tensor rank of the matrix multiplication tensor quantifies the computational cost of multiplying two matrices, the so-called stabilizer rank of a certain tensor is a useful barometer for the classical simulation cost of certain quantum computations. In this work, we present number-theoretic and algebraic-geometric techniques for bounding the stabilizer rank of quantum states. First, we refine a number-theoretic theorem of Moulton to exhibit an explicit sequence of product states with exponential stabilizer rank but constant approximate stabilizer rank, and to provide alternate (and simplified) proofs of the best-known asymptotic lower bounds on stabilizer rank and approximate stabilizer rank, up to a log factor. Second, we find the first non-trivial examples of quantum states with multiplicative stabilizer rank under the tensor product. Third, we introduce and study the generic stabilizer rank using algebraic-geometric techniques.

Pierpaola Santarsiero

Title. *Condition number of a decomposition and Terracini loci, what's next?*

Abstract. Working with tensors coming from applied problems there may occur measurement errors and moreover, working with a machine, one is forced to use non exact arithmetic. Therefore, when running algorithms that involve tensors, our actual input is a perturbed tensor and the error representation we are starting with may be amplified. A way to prevent such errors from increasing is by measuring the sensitivity of the tensor decomposition itself. This brings us to the notion of condition number of a decomposition. The r-th Terracini locus of a space of tensors essentially contains all r-uples of rank-one elements having condition number

equal to infinity. In this talk we will focus on the geometric point of view of this object and we will share some ideas for further developments.

Tim Seynnaeve

Title. *Matrix product states, geometry, and invariant theory*

Abstract. In invariant theory of matrices, one studies polynomial functions in the entries of several matrices that are invariant under simultaneous conjugation. I will give an introduction to the subject, and explain how it is closely related to matrix product states; one-dimensional tensor networks arising in quantum information theory. Finally, I will mention some open problems about matrix product states that I believe can be tackled using this connection.

Luca Sodomaco

Title. *Some open problems about singular vector tuples of tensors and ED degrees of Segre-Veronese varieties*

Abstract. In the first part of the talk, we recall the definition and the main properties of singular vector tuples of tensors and eigenvectors of symmetric tensors. We formalize their connection to the computation of the distance from a given tensor to the algebraic variety X of tensors of rank at most one. A measure of the complexity of this problem is furnished by the Euclidean Distance degree of X. In recent years, there has been remarkable progress on the subject, although several open questions remain unsolved. Some of them will be stated during the talk and are inspired by recent works with Ottaviani, Shahidi, Turatti, and Ventura.

Daniel Stilck França

Title. *Metropolis-Hastings sampling of tensor network states and rapid mixing*

Abstract. Tensor networks are the working horse behind several state-of-the-art algorithms to simulate quantum many-body systems. The main computational bottleneck when employing such methods is the contraction of tensor networks to obtain the expectation values of observables. In this talk, I will discuss how to use the Metropolis-Hastings chain to get a more efficient algorithm to estimate the expectation value of local observables of tensor network states provided a Markov chain is rapidly mixing. Furthermore, we will establish rapid mixing for a large class of matrix product states with periodic boundary conditions. This leads to an almost quadratic improvement in the complexity of computing expectation values of local observables in terms of the bond dimension. Finally, we will discuss possible extensions of the method to tensor networks in higher dimensions and shallow quantum circuits.

Jeroen Zuiddam

Title. *The subrank of random tensors*

Abstract. The subrank (introduced by Strassen in 1987 to study matrix multiplication) measures how much a tensor can be "diagonalized". We determine the subrank of random tensors (generic tensors), establishing a large gap between the subrank on the one hand and other parameters like the slice rank, analytic rank and geometric rank on the other hand. This is joint work with Derksen and Makam.

Contributed Talks

Alimzhan Amanov

Title. *Fundamental invariants of tensors, Latin hypercubes and rectangular Kronecker coefficients*

Abstract. We study polynomial SL-invariants of tensors, mainly focusing on fundamental invariants which are of smallest degrees. In particular, we prove that certain 3-dimensional analogue of the Alon–Tarsi conjecture on Latin cubes considered previously by Burgisser and Ikenmeyer, implies positivity of (generalized) Kronecker coefficients at rectangular partitions and as a result provides values for degree sequences of fundamental invariants.

Cosimo Flavi

Title. *Border rank of powers of ternary quadratic forms*

Abstract. We determine the border rank of each power of any quadratic form in three variables. Since the case of forms of rank 1 and 2 is quite simple, we basically focus on the case of non-degenerate quadratic forms. Considering the quadratic form in n variables $q_n = x_1^2 + ... + x_n^2$, we first determine the apolar ideal of q_n^s for every $s \in \mathbb{N}$, which turns out to be generated by harmonic polynomials of degree $s+1$. By this result, we select for each power a specific ideal contained in the apolar ideal of the form q_3^s and, making use of the recent technique of border apolarity, we prove that its border rank is equal to the rank of its central catalecticant map, that is $(s+1)(s+2)/2$.

Masoud Gharahi

Title. *Persistent Tensors and Multiqudit Entanglement Transformation*

Abstract. We construct a lower bound of the tensor rank for a class of tensors, which we call persistent tensors. Then, we present a specfic subclass of persistent tensors, of which the lower bound is tight. In addition, we show that this subclass of persistent tensors is indeed a generalization of multiqubit W states within multiqudit systems (we call them multiqudit M states) and is geometrically in the orbit closure of multiqudit GHZ states. Consequently, we show that one can obtain a multiqudit M state from a multiqudit GHZ state via asymptotic Stochastic Local Operations and Classical Communication (SLOCC) with rate one. Moreover, we show that the tensor rank of the Kronecker product, and hence the tensor product of multiqudit GHZ and M states is equal to the product of their tensor ranks.

Harshit Motwani

Title. *The linear span of uniform matrix product states*

Abstract. The variety of uniform matrix product states arises both in algebraic geometry as a natural generalization of the Veronese variety, and in quantum many-body physics as a model for a translation-invariant system of sites placed on a ring. Using methods from linear algebra, representation theory, and invariant theory of matrices, we study the linear span of this variety.

Tommi Muller

Title. *Robust Eigenvectors of Symmetric Tensors*

Abstract. The tensor power method generalizes the matrix power method to higher order arrays, or tensors. Like in the matrix case, the fixed points of the tensor power method are the eigenvectors of the tensor. While every real symmetric matrix has an eigendecomposition, the vectors generating a symmetric decomposition of a real symmetric tensor are not always eigenvectors of the tensor. In this paper we show that whenever an eigenvector is a generator of the symmetric decomposition of a symmetric tensor, then (if the order of the tensor is

sufficiently high) this eigenvector is robust, i.e., it is an attracting fixed point of the tensor power method. We exhibit new classes of symmetric tensors whose symmetric decomposition consists of eigenvectors. Generalizing orthogonally decomposable tensors, we consider equiangular tight frame decomposable and equiangular set decomposable tensors. Our main result implies that such tensors can be decomposed using the tensor power method.

Alan Muniz

Title. *Foliations on homogeneous spaces*

Abstract. A codimension-1 foliation on a projective manifold X can be thought of as a corank-1 saturated subsheaf F of the tangent bundle TX which is stable under the Lie bracket. After fixing the determinant of F, the set of such foliations is a locally closed subset of the space of 1-forms on X with values in some fixed line bundle L. We investigate the set of these foliations when X is a projective homogeneous space of Picard rank 1, for the simplest possible choice of L. A special focus will be on Grassmannians, most notably Grassmannians of lines, where our foliations are particularly easy to describe. This is a work in progress with V. Benedetti and D. Faenzi.

Ettore Turatti

Title. *Tensors determined by their singular vector tuples*

Abstract. We will introduce the notion of singular vector tuples and eigenschemes for partially symmetric tensors. Then, we will study the question: given a general tensor t, there exists other tensors that have the same singular tuples of t? We will show that if there exists at least one component of odd degree, then only t has those singular tuples, otherwise there is a 1-dimensional space of tensors with the same singular tuples as t.

Francesca Zaffalon

Title. *Toric degenerations of partial flag varieties via matching fields and combinatorial mutations*

Abstract. Toric degenerations are an important tool that can be used to analyze algebraic varieties. They allow us to understand a general variety via the geometry of their associated toric varieties, since many geometric invariants are preserved via degenerations. In this talk, I will show how to produce a new large family of toric degenerations of Grassmannians and (partial) flag varieties, whose combinatorics is governed by matching fields. Moreover, I will study the relations between polytopes associated to different toric degenerations of the same variety. This is done using the tool of combinatorial mutations, particular piecewise linear functions on polytopes. Finally, I will show how our methods give new families of toric degenerations of small Grassmannians and flag varieties