HYPERGRAPH LSS-IDEALS AND COORDINATE SECTIONS OF SYMMETRIC TENSORS

Let $\mathbb{K}$ be a field, $[n] = \{1, \dots, n\}$ and $H = ([n], E)$ be a hypergraph. For an integer $d \geq 1$ the Lovász-Saks-Schrijver ideal (LSS-ideal) $L_H^{\mathbb{K}}(d) \subseteq \mathbb{K}\big[\, y_{ij} \;:\; (i,j) \in [n] \times [d] \,\big]$ is the ideal generated by the polynomials $f_e^{(d)} = \sum_{j=1}^{d} \prod_{i \in e} y_{ij}$ for edges e of H. For an algebraically closed field $\mathbb{K}$ and a k-uniform hypergraph $H = ([n], E)$ we employ a connection between LSS-ideals and coordinate sections of the closure of the set $S_{n,k}^d$ of homogeneous degree k symmetric tensors in n variables of rank $\leq d$ to derive results on the irreducibility of its coordinate sections. To this end we provide results on primality and the complete intersection property of $L_H^{\mathbb{K}}(d)$. We then use the combinatorial concept of positive matching decomposition of a hypergraph H to provide bounds on when $L_H^{\mathbb{K}}(d)$ turns prime to provide results on the irreducibility of coordinate sections of $S_{n,k}^d$.